Revista Sergipana de Matemática e Educação Matemática

https://doi.org/10.34179/revisem.v7i2.16828

EXTENSION TO THE PEUKERT'S LAW MODEL APPLIED TO THE BATTERY LIFETIME PREDICTION PROBLEM

Douglas J. B. Freitas
Universidade Regional do Noroeste do Estado do Rio Grande do Sul - Unijuí
douglas.joziel@outlook.com

Airam T. Z. R. Sausen
Universidade Regional do Noroeste do Estado do Rio Grande do Sul - Unijuí
airam@unijui.edu.br

Mauricio de Campos
Universidade Regional do Noroeste do Estado do Rio Grande do Sul - Unijuí
campos@unijui.edu.br

Paulo S. Sausen
Universidade Regional do Noroeste do Estado do Rio Grande do Sul - Unijuí
sausen@unijui.edu.br

Abstract

Accurate prediction of the rechargeable battery lifetime is important for the optimization of the use of different types of mobile devices in the present time. Many of them use simple analytical models for this task, such as Peukert's Law model. This paper aims to evaluate the accuracy of battery lifetime prediction, from an extension of Peukert's Law. The original and extended models are evaluated by comparing the results of computational simulations and a wide set of experimental data obtained from a test platform under scenarios of constant and variable current discharges, the extended model is also compared with an accurate model of the literature, i.e., the Rakhmatov and Vrudhula's diffusion model. For lithium-ion polymer batteries, the proposed extended model is accurate, it produced an average error of 1.07 % in the prediction of battery lifetime for discharges with constant current profiles and an average error of 2.57 % for variable discharge profiles that simulate the power drain of a smartphone used for a variety of everyday applications.

Resumo

A predição acurada do tempo de vida de baterias recarregáveis é importante para a otimização do uso de diferentes tipos de dispositivos móveis na atualidade. Muitos dispositivos usam modelos analíticos simples para essa tarefa, como o modelo da Lei de Peukert. Este artigo tem como objetivo avaliar a acurácia da predição do tempo de vida da bateria, a partir de uma extensão à Lei de Peukert. Os modelos original e estendido são avaliados comparando os resultados das simulações computacionais com um amplo conjunto de dados experimentais obtidos de uma plataforma de testes em cenários de descargas de corrente constante e variável, o modelo estendido também é comparado com um modelo acurado da literatura, ou seja, o modelo de difusão de Rakhmatov e Vrudhula. Para baterias de polímero de íon-lítio, o modelo estendido proposto apresentou bons resultados, ele produziu um erro médio de 1,07 % na predição do tempo de vida da bateria para descargas com perfis de corrente constantes e um erro médio de 2,57 % para perfis de descargas variáveis que simulam o consumo de energia de um *smartphone* usado para uma variedade de aplicações diárias.

1 Introduction

The increasing technological evolution that has occurred over the last decade has generated significant changes in commonplace activities, particularly with regard to methods of communication. The user of mobile device needs to be connected to the voice and data network at all times, this fact has contributed to the popularization of mobile devices including smartwearables, cell phones, smartphones, tablets and notebooks, among others. While these devices offer mobility, convenience and ease of use, their operational autonomy is limited by the performance of the power source.

The rechargeable batteries used in mobile devices typically have low power storage capacities and require recharge after each period of use. According to Liu et. al. [11], the rapid recharge of the device depends essentially on an accurate model. However, the ideal model for each device depends mainly on its processing capacity. Therefore, it is important to develop more effective methods for the battery lifetime prediction and, consequently, to know the operating time of mobile devices. Rakhmatov and Vrudhula [18] explain that the lifetime, or time-to-failure, of the battery is the time when it becomes fully discharged, i.e., the battery is exhausted and the mobile device shuts down.

Battery lifetime predictions may be acquired through the use of mathematical models that describe the dynamic behavior of the discharge based on the its actual physical characteristics or on a reduced set of experimental data. Among the various mathematical models that have been described for this purpose, are particularly attractive the models electrochemical [1, 5, 8], electric [1, 2, 8, 14], stochastic [1, 3, 8], analytical [17, 18], system identification [9, 19] or, more recently, a hybrid approach [10]. In general terms, analytical models describe the battery in a more abstract manner, modeling their main properties based on a reduced set of equations. Such models describe either constant or variable current discharges in the time domain, and in many cases capture the nonlinear effects of the process.

One of the simplest analytical models is the Peukert's Law [16]. Although this model is computationally efficient and flexible requiring only the evaluation of analytical expression when set up for different types of batteries [1], it exhibits a greater average error when compared to other more elaborate models. According to Rakhmatov and Vrudhula [17, 18], for example, simulations based on Peukert's Law of the battery lifetime under constant discharge presented an average error of 6% and under variable discharge an average error of 8% when compared with simulations made using the Dualfoil program [5] for Lithium-Ion (Li-Ion) battery.

Doerffel and Sharkh [4] explain that the Peukert's Law cannot be used to accurately predict the remaining capacity and, consequently, the battery lifetime unless it is discharged to a temperature and load constant. On the other hand, according to Muhammad et. al [14] the estimation of the remaining capacity of a battery is very important for many applications. The estimated value of remaining capacity can be used to improve the efficiency of the individual cell as well as to schedule the use of cells in the multicellular system to maximize the discharge time.

Freitas et. al [7] proposed an extension of the original Peukert's Law and presented an initial analysis of the model operation from a set of constant discharge current profiles and only one variable discharge current profile. From a statistical point of view, this analysis is not considered sufficiently robust [12, 13] for the model acceptance, being necessary both a wide inspection considering a larger dataset of variable discharge current profiles and a comparative analysis with an accurate diffusion model [17, 18] from the literature.

There is currently a considerable range of mobile devices with low processing power. Therefore, choosing a mathematical model to be embedded into an application in the device is an important task, because the model must balance the intended accuracy and the computational cost required [6, 15]. In this regard the development of simple and accurate models that can easily be embedded in mobile devices are relevant.

This paper presents the extended model of Peukert's Law, being considered a simple and accurate model that can be used for the battery lifetime prediction. An evaluation of the extended model is carried out in three ways, first a comparative analysis is performed from the simulation results of the battery lifetime by extended model and the wide set of experimental data obtained by discharging Lithium-Ion Polymer (Li-Po) batteries from a testbed, second the extended model is compared with the original Peukert's Law, and finally it is compared with the Rakhmatov and Vrudhula's diffusion model (i.e., RV model) [17, 18], which is one accurate model of the literature for the battery lifetime prediction.

The remainder of this paper is organized as follows. Section 2 presents the algebraic breakdown of the extension to the Peukert's Law, and the equations of the RV model are also presented. The testbed and the methodologies employed to collect experimental data and to estimate the model parameters, more specifically the Least Squares (LS) method, are described in the Section 3, while Section 4 discusses the simulation results and the comparative analysis with experimental data, the original Peukert's Law, and the RV model. Finally, Section 5 presents the conclusions and the perspectives for future work.

2 Mathematical Modeling

By definition, batteries are devices that convert the chemical energy stored into electrical energy by means of electrochemical redox reactions (Fig. 1). In rechargeable batteries, the electron flow through these redox reactions can be reversed when the device is connected to an external power source (i.e., battery).

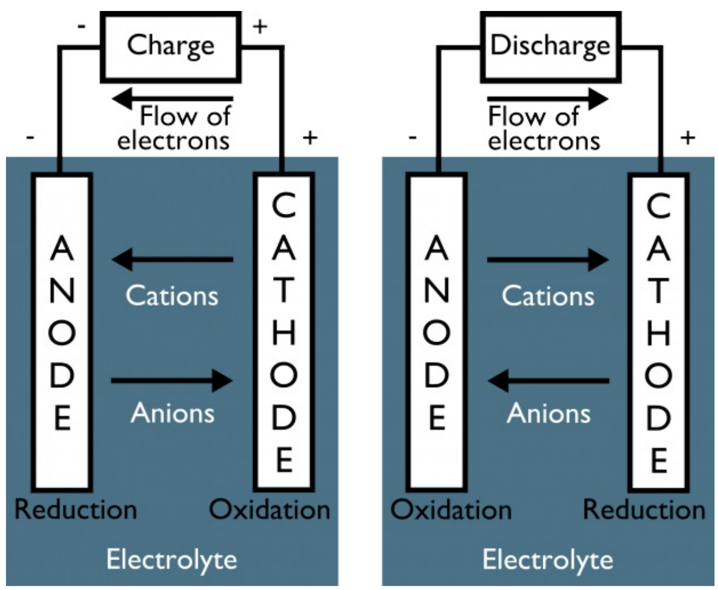

Figure 1: Charge and discharge battery scheme.

In practice, battery discharge is associated with a number of characteristics and nonlinear effects, including cutoff level, recuperation effect and capacity rate [17, 18]. The latter two phenomena are fundamental and must be taken into consideration in mathematical modeling of battery discharge because they directly affect lifetime and energy quantity of the battery. According to Fig. 2, during periods when discharge current is significantly reduced, battery relaxation occurs and there is a reorganization of electrons available. The recuperation effect extends the battery capacity to supply energy before reaching the minimum electrical tension limit (i.e., cutoff level) required to maintain the operational system. On the other hand, the effect of capacity rate depends on the actual capacity of the battery and on the intensity of the discharge current. At high currents, the effective capacity of the battery and its lifetime are reduced because there is insufficient time for the electroactive species to reorganize inside the battery. Both effects are observed more readily in discharges involving currents that are variable in the time domain [9].

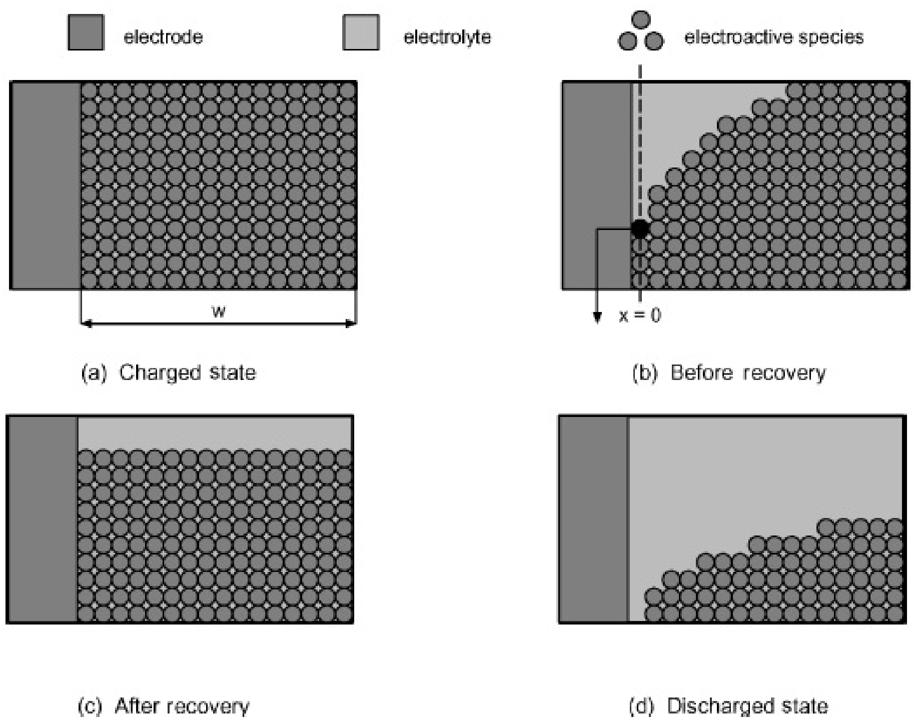

Figure 2: Nonlinear effects.

2.1 Extension of Peukert's Law Model

The original Peukert's Law, proposed by the german engineer Wilhelm Peukert in 1897, is an empirical equation that describes the relationship between the lifetime and the discharge rate of the battery considering only the effect of capacity rate [16]. According to the Peukert equation, the lifetime of a battery ($L > 0$) in hours (h) can be approximated by:

$$L = \frac{a}{I^b}, \tag{2.1}$$

where: $I > 0$ is the discharge current in amperes (A), and a and b are parameters that depend on the type of battery and are estimated from experimental data. In an ideal battery, a would be equal to the capacity of the battery in ampere-hours (Ah) and b would be equal to 1. In practice, however, a has a value that is near to the capacity of the battery and b is a number greater than 1 [16].

Although the model described by equation (2.1) is applicable only to batteries discharged at constant currents, a generalization of the Peukert equation for variable currents can be obtained, according to Rakhmatov and Vrudhula [17, 18], by substituting the discharge current I by the weighted average of the current as a function of time ($t_0 \leq t_k \leq t_n$), as follows:

$$L = \frac{a}{\left[\frac{\sum_{k=1}^{n} I_{k-1}(t_k - t_{k-1})}{t_n}\right]^b}, \tag{2.2}$$

where: I_k is the value of the discharge current at time t_k, and a and b are the parameters defined for equation (2.1). When $n = 1$, equation (2.2) reduces to the original Peukert's Law equation (2.1).

When Peukert's Law is compared with more complex analytical models, for example the RV model, it yields an average error of 6% for discharges with constant currents, and 8% for discharges with variable currents [17]. However, by comparing the 1^{st} and 2^{nd} order derivatives of equation (2.1) with respect to time, it is possible to establish a functional relation $f : I \to L$, such that the average error of Peukert's Law is reduced. Then, rewriting equation (2.1) so that I is a function of L gives:

$$I = \left(\frac{a}{L}\right)^{\frac{1}{b}} \tag{2.3}$$

for which the 1^{st} order derivative with respect to the time L is:

$$\frac{dI}{dL} = -\frac{\left(\frac{a}{L}\right)^{\frac{1}{b}}}{bL}. \tag{2.4}$$

Substituting equation (2.3) in equation (2.4) gives:

$$-L\frac{dI}{dL} - \frac{I}{b} = 0. \tag{2.5}$$

Analogously, the 2^{nd} order derivative of equation (2.3) with respect to the time L is:

$$\frac{d^2I}{dL^2} = \frac{\left(\frac{a}{L}\right)^{\frac{1}{b}}(b+1)}{b^2L^2} \tag{2.6}$$

and substituting equation (2.3) in equation (2.6) gives:

$$L^2\frac{d^2I}{dL^2} - \frac{I(b+1)}{b^2} = 0. \tag{2.7}$$

The comparison of equations (2.5) and (2.7) produces:

$$L^2\frac{d^2I}{dL^2} + L\frac{dI}{dL} - \frac{I}{b^2} = 0, \tag{2.8}$$

which has the form of an Euler-Cauchy 2^{nd} order Ordinary Differential Equation (ODE).

Then the general solution of the equation (2.8) is given by:

$$I = C_1L^{\frac{1}{b}} + C_2L^{-\frac{1}{b}}. \tag{2.9}$$

Rewriting equation (2.9) so that L is a function of I, and conveniently adopting the negative sign for the term under the square root, the following functional relationship is obtained for the extended model of Peukert's Law:

$$L = \left(\frac{I - \sqrt{I^2 - 4C_1C_2}}{2C_1}\right)^b, \tag{2.10}$$

where C_1, C_2 and b are parameters to be determined from an experimental data set, and $I \geq 2\sqrt{C1C2}$. In order to generalize equation (2.2) for discharges with variable currents, the value of I can be substituted by the weighted average of the current as a function of time, namely:

$$I \equiv \left[\frac{\sum_{k=1}^{n} I_{k-1}(t_k - t_{k-1})}{t_n}\right], \tag{2.11}$$

to give:

$$L = \left(\frac{\left[\frac{\sum_{k=1}^{n} I_{k-1}(t_k - t_{k-1})}{t_n}\right] - \sqrt{\Delta}}{2C_1}\right)^b \tag{2.12}$$

and

$$\Delta = \left[\frac{\sum_{k=1}^{n} I_{k-1}(t_k - t_{k-1})}{t_n}\right]^2 - 4C_1C_2, \tag{2.13}$$

where: I_k is the value of the discharge current at time t_k and C_1, C_2 and b are the same parameters as defined for equation (2.10) and determined according to the methodology adopted by Rakhmatov and Vrudhula [17]. When $n = 1$, equation (2.12) reduces to the extended model of Peukert's Law, i.e., equation (2.10).

2.2 RV Model

The RV model describes the process of diffusion of the electroactive species based on Fick's Laws [17, 18] that are expressed by two Partial Differential Equations (EDPs), given by:

$$-J(x,t) = D\frac{\partial C(x,t)}{\partial x}, \tag{2.14}$$

$$\frac{\partial C(x,t)}{\partial t} = D\frac{\partial^2 C(x,t)}{\partial x^2}, \tag{2.15}$$

where: $J(x,t)$ is the flux of species at distance $x \in [0, w]$ of the electrode and at time $t \in [0, L]$, D is the diffusion coefficient, and $C(x,t)$ is the species concentration.

For a fully charged battery, the species concentration is constant through the electrolyte, then the initial condition of the species concentration is given by:

$$C(x,0) = C^*. \tag{2.16}$$

According to Faraday's law, the flow at the left boundary of the diffusion region ($x = 0$) of the electrode is proportional to the discharge current $i(t)$, at the right boundary of the diffusion region ($x = w$) is zero. Thus, for discharge of a current $i(t)$ and time $0 < t < \infty$, border conditions are:

$$\left.\frac{\partial C(x,t)}{\partial x}\right|_{x=0} = \frac{i(t)}{vFAD}, \tag{2.17}$$

$$\left.\frac{\partial C(x,t)}{\partial x}\right|_{x=w} = 0, \tag{2.18}$$

where: A is the surface area of the electrode, F is the constant of Faraday and v is the electrons number [17] in the chemical reactions on the electrode surface.

After the computations presented in [17] the general expression that relates the battery lifetime L and the current discharge $i(\tau)$ is given by:

$$\alpha = \int_0^L \frac{i(\tau)}{\sqrt{L-\tau}}d\tau + 2\sum_{n=1}^{\infty}\int_0^L \frac{i(\tau)}{\sqrt{L-\tau}}e^{-\frac{\beta^2 n^2}{(L-\tau)}}d\tau. \tag{2.19}$$

where: α and β are parameters that need to be estimated.

3 Estimation of Parameters

This section describes a methodology for the estimation of the parameters a and b in the original Peukert's Law model, C_1, C_2 and b in the proposed extended model, and α and β in the RV model. These parameters are specific to each type of battery and, in the present research, relate to eight new PL-383562-2C Li-Po batteries (B_1 to B_8) that are employed in the laboratory tests due to their common use as power sources in mobile devices.

3.1 Testbed

Unlike other literature research [4, 17, 18] that uses simulators for the acquisition of experimental data, in this paper the experimental data were acquired using a testbed, as Figure 3, that make the loading and unloading of Li-Po batteries. The management interface incorporated into the testbed allowed the rapid configuration of experiments, and enabled up to four discharges to be carried out simultaneously with the information for each being stored in a separate file to facilitate data searching. It is constituted of three basic parts: (i) software, (ii) hardware and, (iii) batteries.

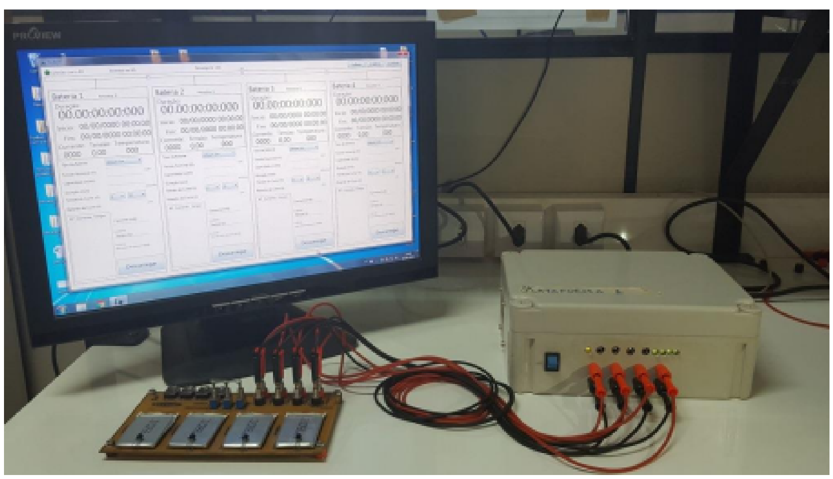

Figure 3: Testbed - unloading procedure

The software is developed in C++, it has an intuitive interface for the information of the batteries parameters, being responsible for sending the configurations of the type of discharge to the hardware, so the results generated by the tests are obtained. After completing data (i.e., current value and discharge type) at the interface, the discharge control to which the batteries are subjected is managed by the software. In addition, the platform has protection resources in case of any problems in the operational system and also allows saving the graphics in bitmap format and reports in text format. In case of serial communication failures between the hardware and the software, or when the battery voltage curve simulated by the platform reaches the cutoff level, the platform stops operating immediately. The hardware communicates the platform with the computer and manages the sensing and download control modules.

3.2 Collection of Discharge Data

All data collections followed the same methodology for charging and discharging batteries in order to reduce any variation in the final test results. Initially various profiles were employed in experiments involving discharge at constant currents in the range 0.05 to 0.8 A, and intervals were maintained between profiles involving low, medium and high intensity currents. Each constant current profile generated an individual discharge time for each battery (B_1 to B_8), and an average lifetime ($\bar{L}$) was calculated. The data obtained were separated into two sets, namely one for parameter estimates (I_{est} and $\bar{L}_{est}$, Table 1), and another for model validation (I_{cval} and $\bar{L}_{cval}$, Table 4) presented in the Section 4 along with the results.

Table 1: Data for the estimation of parameters.

I_{est} (A)	$\bar{L}_{est}$ (h)
0.0500	15.6728
0.2500	3.0668
0.4500	1.6819
0.6000	1.2449
0.8000	0.9107

Following the same methodology, variable discharge profiles are obtained from experimental tests performed on a smartphone, which performs the requested everyday tasks presented in Table 2.

From this experiment eight realistic variable discharge profiles (P_1-P_8) was constructed and presented in the Table 3. Each variable current profile generated an individual discharge time for each battery (B_1 to B_8), and an average lifetime ($\bar{L}_{vval}$) was calculated. The data obtained were presented in the Section 4 along with the results, in the Table 5.

Table 2: Discharge currents related to the tasks performed.

Tasks Description	Discharge (A)
Stand by	0.01
Display on and use of device with minimal brightness	0.04
Display on and using the device with maximum brightness	0.07
Camera usage	0.08
Speakerphone call	0.15
Simple call	0.1
Visualize image	0.1
Listening to music at full volume	0.2
Listen to music at minimum volume	0.1
Access the internet	0.2
Using the calculator	0.05
Write SMS	0.07
Access to device games	0.04
Alarm Usage	0.2
Audio recording	0.09
Listening to music on the radio (full volume)	0.23

Table 3: Variable Discharge Profiles

Profile	Discharge (A)	Time (min)
P1	0.1 - 0.01 - 0.15 - 0.01 - 0.1 - 0.01 - 0.2	5 - 5 - 5 -5 - 5 - 5 - 10
P2	0.17 - 0.27 - 0.01 - 0.14 - 0.23 - 0.01 - 0.27	5 - 20 - 30 - 10 - 20 - 10 - 30
P3	0.27 - 0.01 - 0.12 - 0.17 - 0.01 - 0.27 - 0.17	5 - 10 - 10 - 15 - 10 - 15 - 5
P4	0.25 - 0.4 - 0.05 - 0.2 - 0.55	10 - 10 - 5 - 15 - 10
P5	0.75 - 0.45 - 0.2 - 0.15 - 0.25 - 0.1	5 - 10 - 10 - 5 - 5 - 10
P6	0.1 - 0.2 - 0.3 - 0.4 - 0.5 - 0.6 - 0.7	10 - 10 - 10 - 10 - 10 - 10 - 10
P7	0.7 - 0.6 - 0.5 - 0.4 - 0.3 - 0.2 - 0.1	10 - 10 - 10 - 10 - 10 - 10 - 10
P8	0.2 - 0.02 - 0.3 - 0.02 - 0.2 - 0.02 - 0.4	2.5 - 2.5 - 2.5 - 2.5 - 2.5 - 2.5 - 5

Unlike discharge at constant currents, these profiles vary over time, so nonlinear effects are more present and the discharge process is more true to the profile of tasks performed by a user.

3.3 Least Squares Method

The LS method is an optimization technique that attempts to find optimum parameter values for a particular data set by minimizing the residues squares sum. In this paper, the nonlinear LS method is employed, it is implemented within MatLab algebraic and numeric computation software through the *lsqnonlin* optimization function. The routines to estimate the parameters of the original and extended Peukert's Law models, and RV model were actualized through m-file scripts with the vectors I_{est}, $\bar{L}_{est}$ and $\varepsilon_0 = [0.8, 1]^T$ considered as initial approximation of the original model, $\varepsilon_0 = [0, 0.8, 1]^T$ as initial approximation of the extended model, and $\varepsilon_0 = [4.2, 0.5]^T$ as initial approximation of the RV model. In this manner, values for the parameters a and b were estimated to be 0.7393 and 1.0195 for the Peukert's Law model, while those for C_1, C_2 and b were estimated to be 0.0004, 0.7369 and 1.0445 for the proposed extended model. The RV model parameters α and β were estimated to be 3.1490 and 0.4449.

4 Simulation Analysis

This section presents the simulation results of Li-Po battery lifetime considering the model proposed, extended Peukert's Law. The evaluation of the results was based on the difference between the lifetime simulated by the model and the average experimental lifetime obtained from the testbed for constant and variable currents discharges. For the validation 15 constant discharge profiles (i.e., Table 4) and 8 variable discharge profiles (i.e., Table 5) were considered and the average errors of the model was computed. Finally, a comparative analysis was performed from the simulation results of the battery lifetime by extended model with the original Peukert's Law, and with the RV model, which is one of the most accurate mathematical models in the literature for the battery lifetime prediction.

4.1 Discharge at Constant Current

In order to evaluate the extended Peukert's Law, the original and extended Peukert's Law models and the RV model were validated. The simulation results of the models for discharge at constant current were compared with a set of experimental data presented in the Table 4.

All models described the experimental data satisfactorily, but the proposed extended model presented a more accurate result with an average error of 1.07% compared with 1.41% obtained with the original model and 1.11% with the RV model. Thus, although the proposed model based on an extension of Peukert's Law requires one parameter

Table 4: Data collected and validation of the models for constant current profiles.

Val. data		Peukert's Law		Ext. Peukert's Law		RV Model	
I_{cval} (A)	$\bar{L}_{cval}$ (h)	L_{sim} (h)	E (%)	L_{sim} (h)	E (%)	L_{sim} (h)	E (%)
0.0750	10.1156	10.3687	2.5023	10.3512	2.3289	10.4759	3.5617
0.1250	6.4127	6.1595	3.9490	6.2616	2.3568	6.1259	3.0691
0.1750	4.5372	4.3708	3.6666	4.4460	2.0103	4.4155	2.6825
0.2250	3.3915	3.3829	0.2532	3.4325	1.2085	3.4169	0.7486
0.2750	2.7528	2.7570	1.1513	2.7888	1.3060	2.7836	1.1185
0.3250	2.3548	2.3252	1.2528	2.3449	0.4194	2.3461	0.3690
0.3750	2.0518	2.0096	2.0586	2.0207	1.5153	2.0251	1.3013
0.4250	1.8064	1.7688	2.0777	1.7739	1.7958	1.7795	1.4886
0.4750	1.5710	1.5792	0.5230	1.5799	0.5660	1.5839	0.8211
0.5250	1.4366	1.4260	0.7349	1.4234	0.9166	1.4278	0.6126
0.5750	1.2974	1.2997	0.4186	1.2946	0.2137	1.2972	0.0154
0.6250	1.1888	1.1938	0.1788	1.1868	0.1686	1.1886	0.0168
0.6750	1.0995	1.1037	0.3803	1.0953	0.3876	1.0959	0.3274
0.7250	1.0115	1.0262	1.4444	1.0166	0.4977	1.0154	0.3885
0.7750	0.9439	0.9587	1.5707	0.9482	0.4625	0.9459	0.2066
–		$\bar{E} = 1.4108$		$\bar{E} = 1.0769$		$\bar{E} = 1.1152$	

more for estimation, it offers the advantage of reducing the error by 24% over the original model and 3.4% over the RV model, improving the accuracy for the battery lifetime prediction.

It is noteworthy that the extended model produced even better results with profiles at the higher end of the discharge current range (i.e., $I \geq 0.7250\ A$) showing a 3-fold reduction in error compared with the original model. For all models, however, errors of greater amplitude were concentrated in discharge profiles with lower currents (i.e., $I < 0.2500\ A$). This occurred because batteries exhibit longer discharge times at low currents and, consequently, there is a greater contribution of nonlinear effects that directly influence the lifetime.

4.2 Discharge at Variable Currents

The models original and extended Peukert's Law and RV model were also validated for variable discharge profiles. The simulation results were compared with a set of experimental data presented in the Table 5.

Table 5: Data collected and validation of the models for variable current profiles.

Variable data		Peukert's Law		Ext. Peukert's Law		RV Model	
Profile	$\bar{L}_{vval}$ (h)	L_{sim} (h)	E (%)	L_{sim} (h)	E (%)	L_{sim} (h)	E (%)
P1	7.9954	7.8900	1.3177	7.9133	2.3289	10.4759	3.5617
P2	4.7491	4.4333	6.6496	4.4550	2.3568	6.1259	3.0691
P3	5.3670	5.2700	1.8070	5.3100	2.0103	4.4155	2.6825
P4	2.4898	2.4833	0.2597	2.4850	1.2085	3.4169	0.7486
P5	2.3627	2.3950	1.3658	2.3983	1.3060	2.7836	1.1185
P6	2.1104	2.0600	2.3901	2.0583	0.4194	2.3461	0.3690
P7	1.6420	1.6350	0.4243	2.0583	1.5153	2.0251	1.3013
P8	5.4029	5.4417	0.7177	1.7739	1.7958	1.7795	1.4886
–		$\bar{E}$ = 1.8666		$\bar{E}$ = 1.7718		$\bar{E}$ = 1.7470	

It is observed that all models described the experimental data satisfactorily, the proposed extended model presented a more accurate result with an average error of 1.77% compared with 1.87% obtained by original model. Then, the proposed model has the advantage of reducing the error by 5.1% over the original Peukert's Law model. The RV model has an error of 1.75%, in this case the extended model has an error around 1.4% higher than the RV model, this small difference is overcome by the fact that the extended model has an algebraic complexity lower than the VR model. Even considering the RV model as a benchmark, the extended Peukert's Law presented an average error that was well within acceptable limits in the technical literature (i.e., lower than 5%) and very close to the results achieved by the RV model.

5 Conclusions

This paper proposed an extension to the Peukert's Law for prediction of the Li-Po batteries lifetime used in mobile devices. It was found to be functional, simple and computationally flexible. Moreover, it was able to capture the nonlinear functional relationship between battery lifetime and discharge rate. Although the present research was carried out using Li-Po batteries, this new model could be configured for different types of batteries. The functional form of the model was derived by comparing the 1^{st} and 2^{nd} order derivatives of the Peukert's Law model and its empirical parameters were fitted by LS method, based on simulation results. The proposed model were compared with experimental data and simulations results of the original Peukert's Law and RV model under the constant and variable discharges from a testbed. The average error of extended model for discharge at constant currents was 1.41 % and variable currents was 1.86 %.

References

[1] Bartlett, Alexander; Marcicki, James; Onori, Simona; Rizzoni, Giorgio; Yang, Xiao G.; Miller, Ted: Electrochemical model-based state of charge and capacity estimation for a composite electrode lithium-ion battery. IEEE Transactions on Control Systems Technology, **24** (2016), no. 2, 384 – 399.

[2] Brivio, Claudio; Musolino, Vincenzo; Merlo, Marco; Ballif, Christophe: A physically-based electrical model for lithium-ion cells. IEEE Transactions on Energy Conversion, **34** (2019), no. 2, 594 – 603.

[3] Chiasserini, Carla F.; Rao, Ramesh R.: Pulsed battery discharge in communication devices. In Proceedings of the 5th Annual ACM/IEEE International Conference on Mobile Computing and Networking. New York, NY, USA: ACM, 1999, 88–95

[4] Doerffel, Dennis; Sharkh, Suleiman: A critical review of using the peukert equation for determining the remaining capacity of lead-acid and lithium ion batteries. Journal of Power Sources, **155** (2006), no.2, 395 – 400.

[5] Doyle, Marc; Fuller, Thomas F.; Newman, John: Modeling of galvanostatic charge and discharge of the lithium/polymer/insertion cell. Journal of The Electrochemical Society, **140** (1993), no. 6, 1526 – 1533.

[6] Fotouhi, Abbas; Auger, Daniel J.; Propp, Karsten; Longo, Stefano; Wild, Mark: A review on electric vehicle battery modelling: From lithium-ion toward lithium-sulphur. Renewable and Sustainable Energy Reviews, **56** (2016), no. 1, 1008 – 1021.

[7] Freitas, Douglas Joziel Bitencourt; Sausen, Paulo Sérgio; Sausen, Airam Tereza Zago Romcy; Reimbold, Manuel Martín Pérez: Predição do Tempo de Vida de

Baterias: Proposição de uma Extensão à Lei de Peukert. Revista Espacios, **38** (2017), no. 18, 1 – 13.

[8] Gu, Wa B.; Wang, Chaoyang Y.: Thermal-electrochemical modeling of battery systems. Journal of The Electrochemical Society, **147** (2000), no. 8, 2910 – 2922.

[9] Hu, Yiran; Wang, Yue Y.: Two time-scaled battery model identification with application to battery state estimation. IEEE Transactions on Control Systems Technology, **23** (2015), no. 3, 1180-1188.

[10] Kim, Taesic; Qiao, W.: A hybrid battery model capable of capturing dynamic circuit characteristics and nonlinear capacity effects. IEEE Transactions on Energy Conversion, **26** (2011), no. 4, 1172-1180.

[11] Liu, Kailong; Zou, Changfu K.; Li, Kang; Wik, Torsten: Charging pattern optimization for lithium-ion batteries with an electrothermal-aging model. IEEE Transactions on Industrial Informatics, **14** (2018), no. 12, 5463 5474.

[12] Ljung, Lennart. *System Identification: Theory for the User*, Lennart Ljung. 2.ed. Prentice Hall, 1999.

[13] Gujarati, Damodar; Porter, Dawn. *Basic Econometrics*, Damodar Gujarati, Dawn Porter. 5.ed. Irwin/McGraw-Hill, 2008.

[14] Muhammad, Shaheer; Rafique, Muhammad R.; Li, Shuai; Shao, Zili; Wang, Qixin; Guan, Nan: A robust algorithm for state-of-charge estimation with gain optimization. IEEE Transactions on Industrial Informatics, **13** (2017), no. 6, 2983-2994.

[15] Pattipati, Bharath; Sankavaram, Chaitanya; Pattipati, Krishna: System identification and estimation framework for pivotal automotive battery management system characteri stics. IEEE Transactions on Industrial Informatics, **41** (2011), no. 6, 869–884.

[16] Peukert, Wilhelm: "Uber die abhängigkeit der kapazität von der entladestromstärke bei bleiakkumulatoren. Elektrotechnische Zeitschrift, **1** (1897), no. 20, 287-2188.

[17] Rakhmatov, Daler; Vrudhula, Sarma: An analytical high-level battery model for use in energy management of portable electronic systems. In Proceedings of the 2001 IEEE/ACM International Conference on Computer-Aided Design. Piscataway, NJ, USA: IEEE Press, 2001, pp. 488–493

[18] Rakhmatov, Daler; Vrudhula, Sarma; Wallach, Deborah A.: Battery lifetime prediction for energy-aware computing. In Proceedings of the 2002 International Symposium on Low Power Electronics and Design. New York, NY, USA: ACM, 2002, 154–159

[19] Sitterly, Mark; Wang, Le Y.; Yin, George G; Wang, Caisheng: Enhanced identification of battery models for real-time battery managemen. IEEE Transactions on Sustainable Energy, **2** (2011), no. 3, 300-308.